# Missouri Mathematics Core Academic Standards

## Shift Three: Rigor

Cindy Bryant
Director of Mathematics

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION™

# Core Academic Standards (CAS) Mathematics Shifts

Shift 1: **FOCUS**

Shift 2: **COHERENCE**

Shift 3: **RIGOR**

- **Conceptual Understanding**
- **Fluency**
- **Applications**

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION

# Session Overview

- Shift Three
- Rationale for Shift Three
- Curriculum Implications
- Resources
- Additional Information

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION

# Mathematics CAS Shift 3: **RIGOR**

## A. Conceptual Understanding

A conceptual approach to learning mathematics helps students develop depth of mathematical understanding by connecting meaning to procedures.

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION

# Mathematics CAS Shift 3: **RIGOR**

## A. Conceptual Understanding

- Teach more than "how to get the answer" and instead support students' ability to access concepts from a number of perspectives.
- Students are able to see math as more than a set of mnemonics or discrete procedures.
- Conceptual understanding supports the other aspects of rigor (fluency and application)

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION

**Conceptual knowledge** of mathematics consists of logical relationships constructed internally and existing in the mind as a network of ideas … By its very nature, conceptual knowledge is knowledge that is understood.

John Van De Walle (2004)

*Elementary and Middle School Mathematics Teaching Developmentally (Sixth Edition)* *ISBN 0-205-48392-5*

# Conceptual Understanding

The ***Progressions*** *documents provide resources that* aid in…

- developing conceptual understanding often building on children's informal knowledge
- support conceptual knowledge and develop informal strategies to solve problems within the domain
- refining the informal strategies to develop fluency with standard procedures

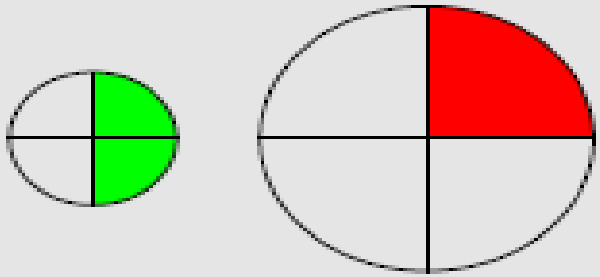
http://ime.math.arizona.edu/progressions/

# Conceptual Understanding

3.NF.3d Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

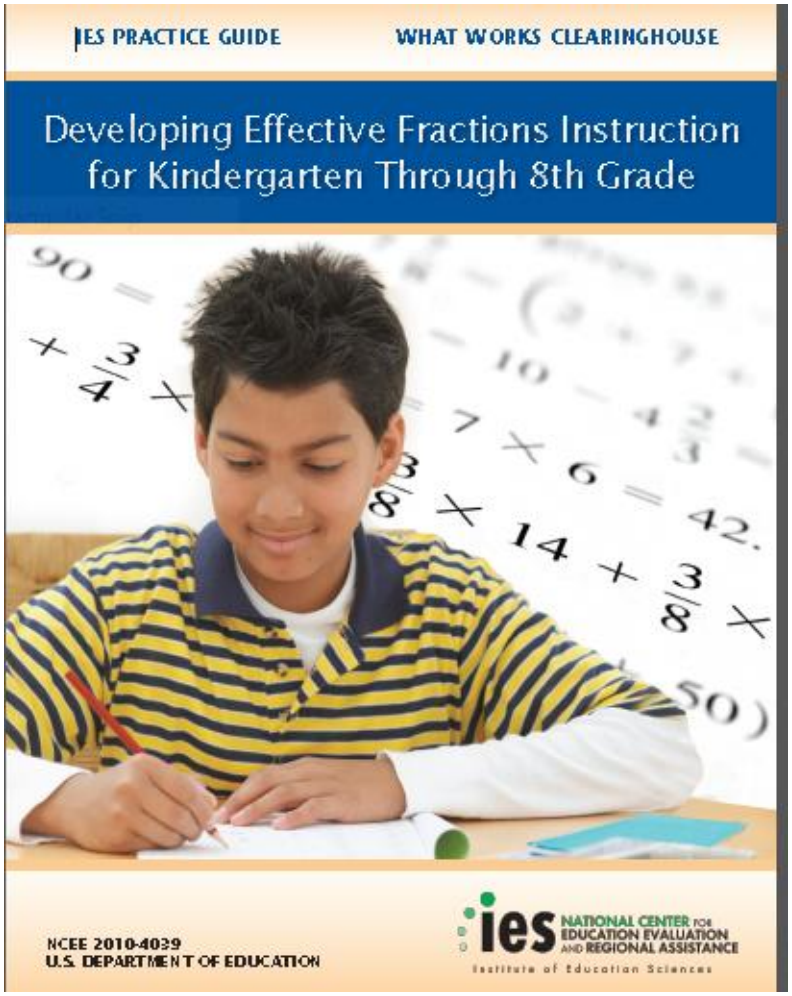
The importance of referring to the same whole when comparing fractions



*A student might think that $\frac{1}{4} > \frac{1}{2}$, because a fourth of the pizza on the right is bigger than a half of the pizza on the left.*

A snapshot from the *Progressions* documents…

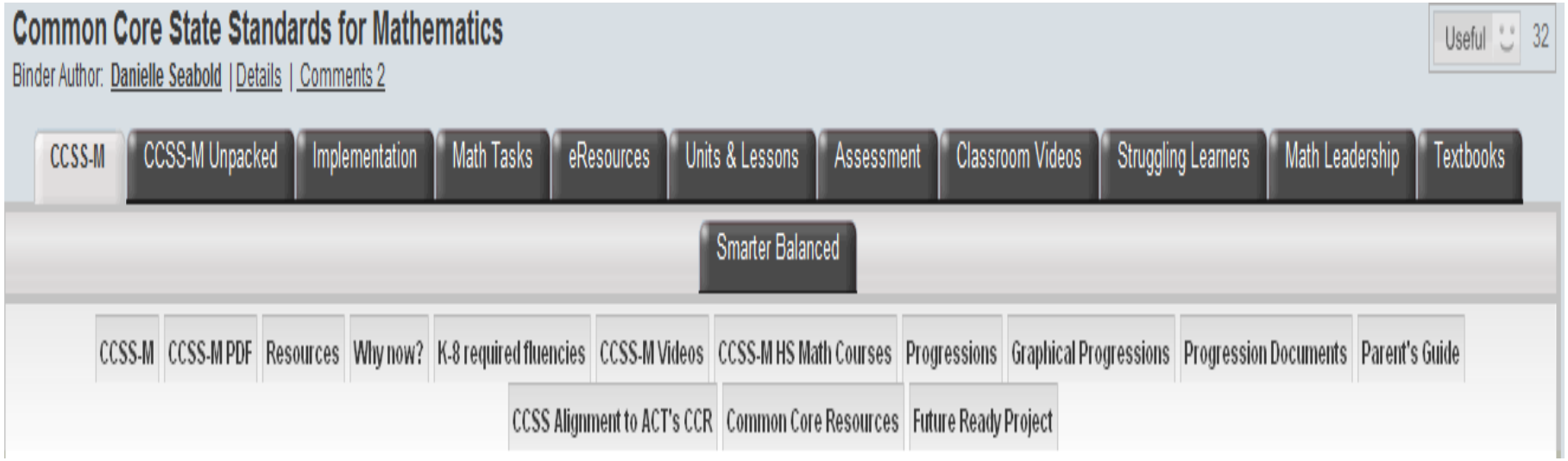
# K – 8 Fraction Instruction




http://ies.ed.gov/ncee/wwc/pdf/practice_guides/fractions_pg_093010.pdf

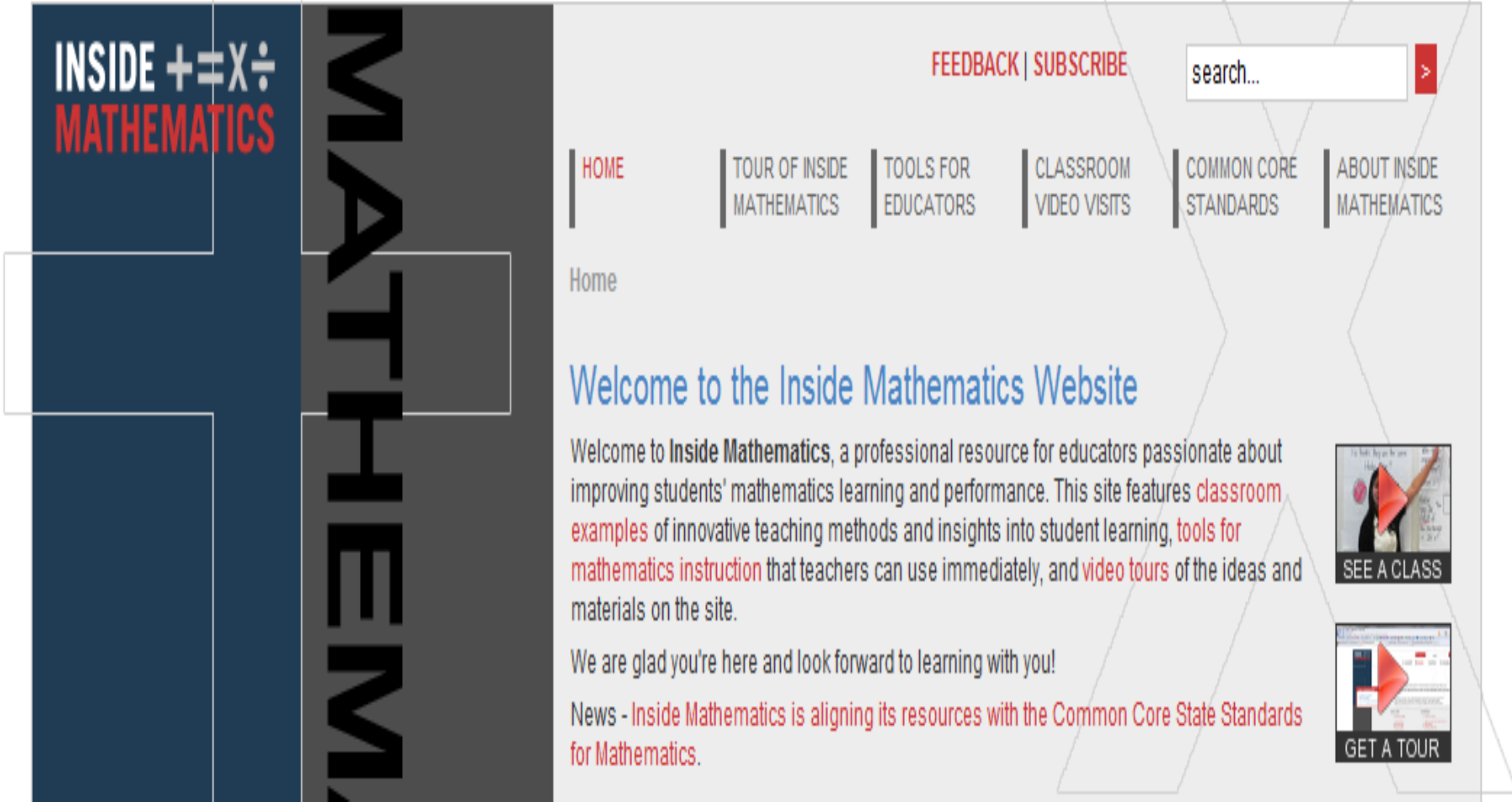
# Common Core LiveBinders Resources

http://www.livebinders.com/play/play/187117



# Inside Mathematics

http://insidemathematics.org/



# Mathematics CAS Shift 3: **RIGOR**

## B. Fluency

*Fluent* in the Standards means "fast and accurate." It might also help to think of fluency as meaning the same thing as when we say that somebody is fluent in a foreign language: when you're fluent, you flow. Fluent isn't halting, stumbling, or reversing oneself.

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION

# Mathematics CAS Shift 3: **RIGOR**

## B. Fluency

The word *fluency* was used judiciously in the Standards to mark the endpoints of progressions that begin with solid underpinnings and then pass upward through stages of growing maturity.

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION

# Required Computational Fluencies in K-6

| Grade | Standard | Required Fluency |
|---|---|---|
| K | **K.OA.5** | Add/subtract within 5 |
| 1 | **1.OA.6** | Add/subtract within 10 |
| 2 | **2.OA.2**<br>**2.NBT.5** | Add/subtract within 20 (know single-digit sums from memory)<br>Add/subtract within 100 |
| 3 | **3.OA.7**<br>**3.NBT.2** | Multiply/divide within 100 (know single-digit products from memory)<br>Add/subtract within 1000 |
| 4 | **4.NBT.4** | Add/subtract within 1,000,000 |
| 5 | **5.NBT.5** | Multi-digit multiplication |
| 6 | **6.NS.2,3** | Multi-digit division<br>Multi-digit decimal operations |

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION

# Mathematics CAS Shift 3: **RIGOR**

## B. Fluency

Conceptual understanding develops fluency.

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION

# Fluency and Standard Algorithms

**Although the Standards do state that students should know standard algorithms, they do not explicitly require or forbid the use of alternate algorithms, which are often strategies based on place value and the properties of operations.**

Standard Algorithm

$$\begin{array}{r} {\scriptstyle 1\phantom{0}} \\ 46 \\ +37 \\ \hline 83 \end{array}$$

Alternate Algorithm

$$\begin{array}{r} 46 \\ +\ 37 \\ \hline \end{array}$$

$$70 + 13 = 83$$

# Mathematics CAS Shift 3: **RIGOR**

## C. Applications

Applications provide an avenue for demonstrating conceptual understanding.

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION


1.
Make sense of problems
and persevere in solving them.
2.
Reason abstractly
and quantitatively.
3.
Construct viable arguments and
critique the reasoning of others.
4.
Model with
mathematics.
5.
Use appropriate tools
strategically.
6.
Attend to
precision.
7.
Look for and make
use of structure.
8.
Look for and express regularity
in repeated reasoning.
K - 12
CAS Standards
for
Mathematical
Practice
Graphic Designed by Kathy Anderson

# *The Standards for Mathematical Practice*

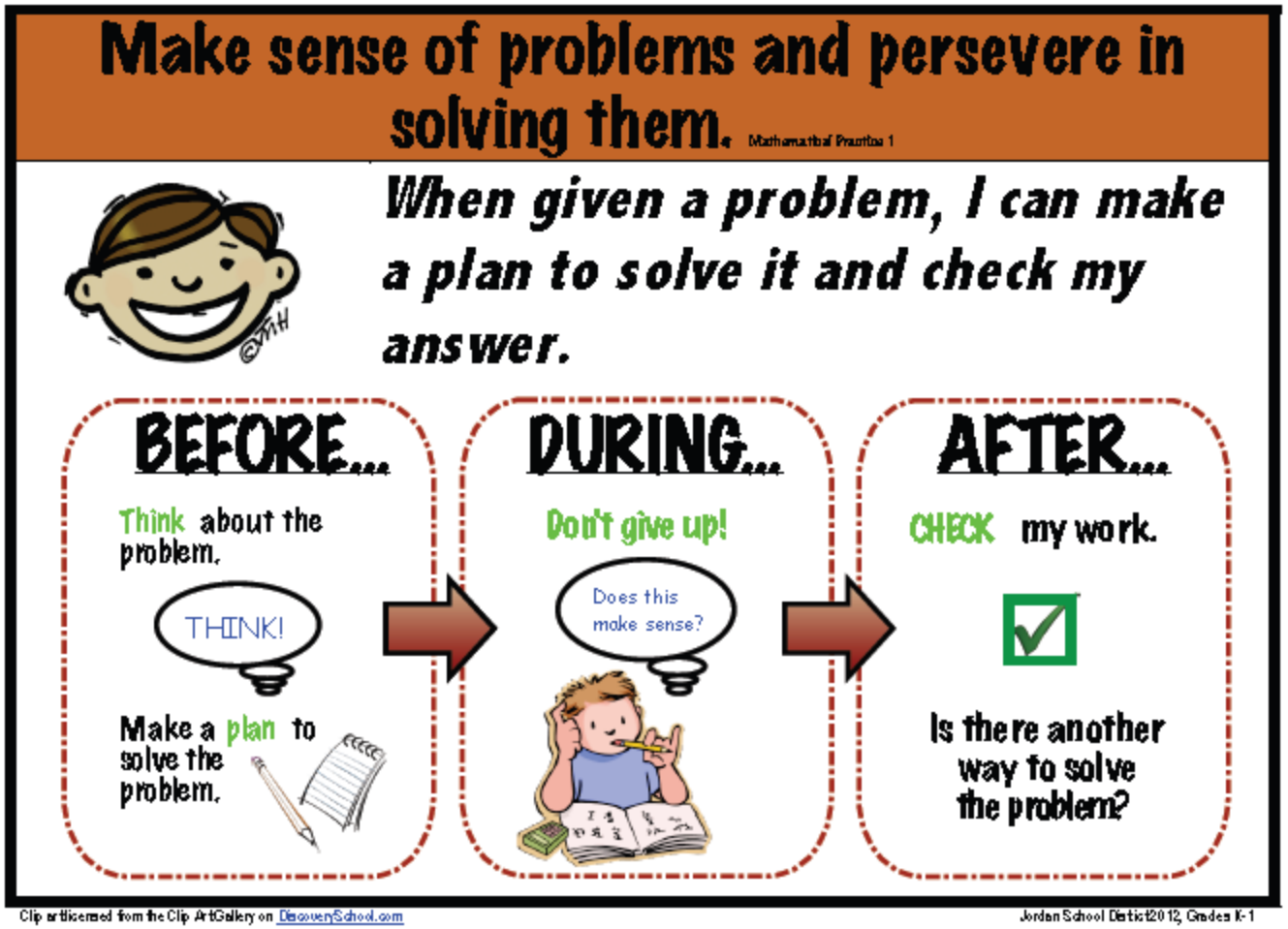
| | | | |
|---|---|---|---|
| 1. Make sense of problems and persevere in problem solving. (3.1 – 3.7) | 2. Reason abstractly and quantitatively. (1.7, 1.10, 3.5, 3.8) | 3. Construct viable arguments and critique the reasoning of others. (1.7,1.8, 3.3, 3.5) | 4. Model with mathematics. (1.6,1.8, 1.10,2.1 3.3 4.1) |
| 5. Us appropriate tools strategically. (1.4, 1.10, 2.7) | 6. Attend to precision. (1.7, 2.2, 2.3, 3.8) | 7. Look for and make use of structure. (1.6, 1.7, 1.8, 2.3, 3.1, 3.6) | 8. Look for and express regularity in repeated reasoning. (1.6, 3.5, 3.6, 3.7) |



The Standards for Mathematical Practice align to the **Missouri Show-Me Process Standards** and provide many opportunities for students to engage in and apply mathematics.

# Mathematical Practices Posters

http://elemmath.jordandistrict.org/mathematical-practices-by-standard/



How would students solve this problem?

Compare the two fractional numbers?

$$\frac{2}{5} \qquad \frac{1}{2}$$

How would students solve this problem?

Compare the two fractional numbers?

$$\frac{2}{5} = \frac{4}{10} \qquad \frac{1}{2} = \frac{5}{10}$$

# How would students solve this problem?
# Compare the two fractional numbers?

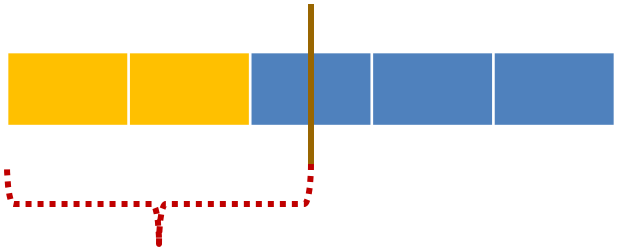
**Same size wholes**

2 of 5

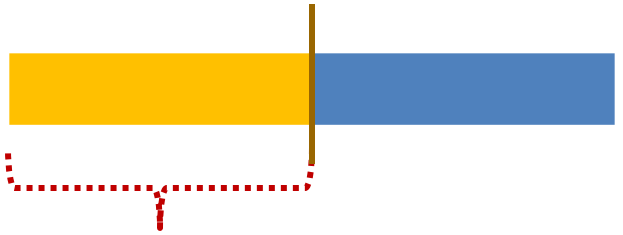
1 of 2

# How would students solve this problem?

# Compare the two fractional numbers?



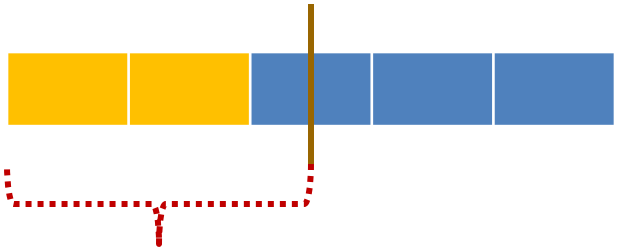
2 is less than half of five equal parts



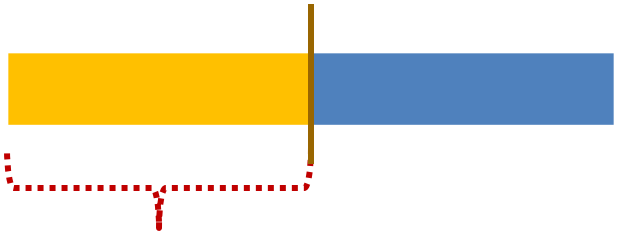
1 is equal to half of two equal parts

# How would students solve this problem?

# Compare the two fractional numbers?



2 is less than half of 5



1 is equal to half of 2

$$\frac{2}{5} < \frac{1}{2}$$

# Mathematics Model Curriculum Units

Provide examples of instructional strategies and activities that:

- Align to the CAS content standards
- Align to the CAS Standards for Mathematical Practice (MP)
- Align to the *Progressions*

https://k12apps.dese.mo.gov/webapps/ModelCurriculum/findunit.aspx

DESE Model Curriculum Pilot Information
https://www.surveymonkey.com/s/8RJBDNM

# Resources

- DESE Mathematics Core Academic Standards Resources
http://dese.mo.gov/divimprove/curriculum/common-core-math.htm
- DESE Mathematics Model Curriculum
https://k12apps.dese.mo.gov/webapps/ModelCurriculum/findunit.aspx
- DESE Model Curriculum Pilot Information
https://www.surveymonkey.com/s/8RJBDNM
- National Council of Teachers of Mathematics
www.nctm.org
- Mathematical Practices Posters
http://elemmath.jordandistrict.org/mathematical-practices-by-standard/
- *Progressions*
http://ime.math.arizona.edu/progressions/

# Mathematics CAS Professional Learning Series Certified Trainers

Kim Amsden, Farmington R-7 kamsden@farmington.k12.mo.us
Alan Bancroft, Central RPDC wbancroft@ucmo.edu
Margaret Bangerter, Northwest RPDC bangert@nwmissouri.edu
Cathy Battles, Kansas City RPDC battlesc@umkc.edu
Nancy Bergfeld, Missouri Council of Teachers of Mathematics nbergfeld@sbcglobal.net
Karla Bond, Ft. Zumwalt kbond@fz.k12.mo.us
Myra Collins, Northeast RPDC mcollins@truman.edu
Denise Corio, Grandview R-II coriod@grandviewr2.org
Terri Doman, Southwest Center tdoman@wcr7.org
Susan German, Hallsville sgerman@hallsville.org
Diane Gillaspie, SuccessLink diane@successlink.org
Trish Goddard, Southwest RPDC goddard@missouristate.edu
Bev Kohzadi, Central RPDC kohzadi@ucmo.edu
Linda Null, Southeast RPDC lnull@semo.edu
Ryan Richardson, Morgan County R-II richardson@mcr2.k12.mo.us
Christina Sheffel, South Central RPDC, sheffelc@mst.edu
Linda Shippy Central RPDC lshippy@ucmo.edu
Pearl Thompson, St. Louis Charter School pthompson@stlcharterschool.org
Melanie Trentmann, Washington melanie.trentmann@washington.k12.mo.us
Melissa Ziegler, Washington melissa.ziegler@washington.k12.mo.us

# Contact Information

Cindy Bryant

Director of Mathematics

Cindy.Bryant@dese.mo.gov

Subscribe to the Curriculum Listserv at:
http://www.dese.mo.gov/divimprove/curriculum/curriclistserv_subscribe.htm

Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION